tification of high-energy perturbation theory, as well as an account of the various problems to which it has been applied, see R. Eden, P. Landshoff, D. Olive, and J. Polkinghorne, The Analytic S-Matrix (Cambridge University Press, Cambridge, England, 1966), Chap. 3.

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[8]This possibility was first suggested by Professor L. F. Cook.

PROOF THAT THE NEAR-FORWARD MINIMUM AND SECONDARY PEAK IN π^-p ELASTIC SCATTERING ARE RESONANCE EFFECTS*

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It is deduced using a very general and simple approach that the differential cross-section minimum near the forward direction and secondary diffraction peak in π^-p elastic scattering in the region from 1.7 to 2.5 BeV/c are resonance phenomena. Experiments of simple interpretation are proposed to determine if this is the general nature of the dip-secondary-peak sequence observed in various reactions.

In a recent Letter, Frautschi[1] suggested that the minima of the differential cross section in the reactions $\pi^\pm+p\rightarrow\pi^\pm+p$ near $t=-0.6$ (BeV)^2 are due to the passage of the P', T_8, and ρ trajectories through a zero near this value of squared momentum transfer in conjunction with the existence of Chew's "ghost-killing" mechanism[2] for the 2^+ nonet. Under these conditions the helicity-flip amplitude (in the t channel) is expected to vanish in this t region and thus give rise to a minimum in the differential cross section in agreement with experiment. Frautschi considered the fact that the polarization changes sign in that vicinity at 2.1 BeV/c to be a confirmation of his ideas.[3]

In the present note we wish to present the results of an analysis of the π^-p elastic differential cross section and polarization in the region from 1.7 to 2.5 BeV/c in the spirit of a very general method we recently proposed.[4,5] According to our results the near-forward minimum of the cross section, the related change of sign of the polarization, and the secondary maximum in this reaction are due to the presence of a resonant amplitude.[6]

Our method is based on the following two considerations:

(1) In the energy region of a resonance, any set of amplitudes that determine a given process may be written in all generality as the sum of two terms: the resonant term plus "the rest," which from now on we will simply call "background." This decomposition has the advantage over the classical partial-wave decomposition that the resonant and background contributions behave quite differently as a function of energy in the region in consideration. Similarly, if there is more than one resonant amplitude contributing to a given enhancement, we may separate them from "the rest."

(2) The behavior of the phase and magnitude of the contribution from a resonant eigenstate to a partial-wave amplitude as functions of the energy is expected to be satisfactorily described by a Breit-Wigner form.[7] Therefore, in pseudoscalar meson–spin-$\frac{1}{2}$ baryon elastic scattering, we may write

$$\tan\delta_l^{\,J}=\Gamma_l^{\,J}/2[(W_R)_l^{\,J}-W], \tag{1}$$

$$|f_l^{\,J}|=x_l^{\,J}\sin\delta_l^{\,J}/k, \tag{2}$$

where f_l^J is the resonant eigenstate contribution to the partial-wave amplitude of orbital (total) angular momentum l (J), δ_l^J its phase or the eigenphase, Γ_l^J the total width, x_l^J the elasticity, $(W_R)_l^J$ the resonant energy, and k the c.m. momentum.

Although only the existence of one resonance [$N^*(2190)$ of spin-parity $\frac{7}{2}^-$] in the vicinity of 2.07 BeV/c has been established,[5] as Yokosawa et al. have reported that there might be at least one other resonating partial wave near this energy we prefer not to ignore a priori the possibility of several resonant partial waves in our analysis. We assume, however, that if there is more than one partial wave contributing to the peak in the cross section, the resonant energies and total widths are the same; the elasticities are allowed to be different. The reason for the constraints is that the simple expressions we are about to derive [Eqs. (6) and (7)] can be obtained only if (a) there is only one resonance contributing to the enhancement, or (b) all the contributing resonances have the same position and width.[8] These formulas should apply approximately if either of these conditions is approximately satisfied.[9]

We choose as our set of amplitudes the spin-nonflip (a) and spin-flip (b) and write them as the sum of two terms,

$$a=a_b+a_r,$$
$$b=b_b+b_r, \tag{3}$$

where

$$a_r=\sum f_l^J(J+\tfrac{1}{2})P_l(\cos\theta),$$

$$b_r=\sum f_l^J(-1)^{J-l+\frac{1}{2}}P_l'(\theta),$$

and the phase and magnitude of each f_l^J are given by Eqs. (1) and (2) with the constraints that all Γ_l^J and $(W_R)_l^J$ (and therefore all δ_l^J) are the same. A standard calculation gives for the differential cross section ($d\sigma/d\Omega$) and the polarization (P)[4] the expressions

$$\frac{d\sigma}{d\Omega}=\left(\frac{d\sigma}{d\Omega}\right)_b+\left(\frac{d\sigma}{d\Omega}\right)_i+\left(\frac{d\sigma}{d\Omega}\right)_r, \tag{4}$$

where

$$(d\sigma/d\Omega)_b=|a_b|^2+|b_b|^2,$$

$$\left(\frac{d\sigma}{d\Omega}\right)_i=2\frac{\sin\delta}{k}\sum_{(l,J)}x_l^J\{|a_b|\cos(\alpha-\delta)(J+\tfrac{1}{2})P_l(\cos\theta)+|b_b|\cos(\beta-\delta)(-1)^{J-l+\frac{1}{2}}P_l'(\theta)\},$$

$$\left(\frac{d\sigma}{d\Omega}\right)_r=\frac{\sin^2\delta}{k^2}\sum_{(l,J)}\sum_{(l',J')}x_l^Jx_{l'}^{J'}F_{(l,J),(l',J')}(\cos\theta),$$

and

$$P\frac{d\sigma}{d\Omega}=\left(P\frac{d\sigma}{d\Omega}\right)_b+\left(P\frac{d\sigma}{d\Omega}\right)_i, \tag{5}$$

where

$$(Pd\sigma/d\Omega)_b=2|a_b|\,|b_b|\sin(\alpha-\beta),$$

$$\left(P\frac{d\sigma}{d\Omega}\right)_i=2\frac{\sin\delta}{k}\sum_{(l,J)}x_l^J\{|a_b|\sin(\alpha-\delta)(-1)^{J-l+\frac{1}{2}}P_l'(\theta)-|b_b|\sin(\beta-\delta)(J+\tfrac{1}{2})P_l(\cos\theta)\}.$$

In these formulas, $P_l'(\theta)$ is the derivative with respect to θ (scattering angle) of the Legendre polynomial $P_l(\cos\theta)$ and α (β) the phase of the amplitude a_b (b_b). The subscripts b, r, and i stand for background, resonance, and background-resonance interference contributions, respectively. The functions $F_{(l,J),(l',J')}(\cos\theta)$ may be easily calculated and are of no special interest in what follows.

We now introduce two quite reasonable assumptions: (1) The phases α and β at any fixed angle vary so slowly with energy in the interval $W_R-\frac{1}{2}\Gamma\leq W\leq W_R+\frac{1}{2}\Gamma$ that they may be approximated by constants; (2) the magnitudes of a_b and b_b at any fixed angle have an energy behavior which does not depart significantly from a k^{-1} dependence in the same interval. (No assumption about the angular dependence of the phases and magnitudes is necessary.) Under these conditions the following relevant fixed-angle relationships are obtained:

$$\left[k^2\left(\frac{d\sigma}{d\Omega}\right)_{i+r}\right]_{\delta=\frac{1}{4}\pi}=\left(k^2\frac{d\sigma}{d\Omega}\right)_{\delta=\frac{1}{2}\pi}-\left(k^2\frac{d\sigma}{d\Omega}\right)_{\delta=\frac{3}{4}\pi}, \tag{6}$$

$$\left[k^2\left(\frac{d\sigma}{d\Omega}\right)_{i+r}\right]_{\delta=\frac{3}{4}\pi}=\left(k^2\frac{d\sigma}{d\Omega}\right)_{\delta=\frac{1}{2}\pi}-\left(k^2\frac{d\sigma}{d\Omega}\right)_{\delta=\frac{1}{4}\pi},$$

$$k^2\left(\frac{d\sigma}{d\Omega}\right)_b = \left(k^2\frac{d\sigma}{d\Omega}\right)_{\delta=\frac{1}{4}\pi} + \left(k^2\frac{d\sigma}{d\Omega}\right)_{\delta=\frac{3}{4}\pi} - \left(k^2\frac{d\sigma}{d\Omega}\right)_{\delta=\frac{1}{2}\pi},$$
$$\left[k^2\left(P\frac{d\sigma}{d\Omega}\right)_i\right]_{\delta=\frac{1}{4}\pi} = \left(k^2P\frac{d\sigma}{d\Omega}\right)_{\delta=\frac{1}{2}\pi} - \left(k^2P\frac{d\sigma}{d\Omega}\right)_{\delta=\frac{3}{4}\pi}, \tag{7}$$
$$\left[k^2\left(P\frac{d\sigma}{d\Omega}\right)_i\right]_{\delta=\frac{3}{4}\pi} = \left(k^2P\frac{d\sigma}{d\Omega}\right)_{\delta=\frac{1}{2}\pi} - \left(k^2P\frac{d\sigma}{d\Omega}\right)_{\delta=\frac{1}{4}\pi},$$
$$k^2\left(P\frac{d\sigma}{d\Omega}\right)_b = \left(k^2P\frac{d\sigma}{d\Omega}\right)_{\delta=\frac{1}{4}\pi} + \left(k^2P\frac{d\sigma}{d\Omega}\right)_{\delta=\frac{3}{4}\pi} - \left(k^2P\frac{d\sigma}{d\Omega}\right)_{\delta=\frac{1}{2}\pi}.$$

By means of these simple equations,[10] we can easily extract from the data the resonance plus interference (interference) contribution to $d\sigma/d\Omega$ $[P(d\sigma/d\Omega)]$ and the background contributions.

We have calculated $k^2(d\sigma/d\Omega)_{i+r}$ at 1.7 BeV/c $(W=W_R-\frac{1}{2}\Gamma)$ using as our input the experimental values of $d\sigma/d\Omega$ at the resonance energy (2.07 BeV/c) and at $W=W_R+\frac{1}{2}\Gamma$ (2.5 BeV/c). We have also calculated this contribution at 2.5 BeV/c from the experimental data at 1.7 and 2.07 BeV/c, and the background contribution, $k^2(d\sigma/d\Omega)_b$. The results are given in Fig. 1.[11,12] One immediately notices that while the background contribution appears to be monotonically increasing, the resonance plus interference contribution exhibits maxima and minima at the "right" values of $\cos\theta$. In fact, the minimum of $(d\sigma/d\Omega)_{i+r}$ near the forward direction moves with increasing energy in the same fashion as the minimum of $d\sigma/d\Omega$.

The results of similar calculations for $k^2[P(d\sigma/d\Omega)]_i$ at 1.7 and 2.5 BeV/c have been plotted in Figs. 2(a) and 2(b). It is seen that in the region $\cos\theta<0.8$ $[-t>0.2$ (BeV)^2 at 1.7 BeV/c and $-t>0.4$ (BeV)^2 at 2.5 BeV/c], $[P(d\sigma/d\Omega)]_i$ reproduces quite well all the features of $P(d\sigma/d\Omega)$. In particular, it reproduces the change of sign near $t=-0.6$ (BeV)^2. In contrast, the background contribution [plotted in Fig. 2(c)] appears to be negative for all values of $\cos\theta$ (i.e., it does not change sign) in the region of interest.[13]

We find these results very striking. Only a severe breakdown of the conditions stated in our assumptions could give rise to these results if Frautschi's suggestion was correct.

That our assumptions are adequate and our results valid seems to be strongly indicated by the fact that our plots show the characteristic behavior of a $G_{7/2}$ resonance (which seems to be the dominant resonance in this region) interfering with a predominantly positive-imaginary nonoscillatory spin-nonflip background (with an out-of-phase spin-flip part): (1) The stationary values and zeros of $P(d\sigma/d\Omega)_i$ fall "close" to the stationary values and zeros of the function $P_4'(\theta)$, (2) $P(d\sigma/d\Omega)_i$ resembles $P_4'(\theta)$ at $W=W_R-\frac{1}{2}\Gamma$ and $-P_4'(\theta)$, plotted in Fig. 2(b), at $W=W_R+\frac{1}{2}\Gamma$,[14] and (3) the maxima and minima of $(d\sigma/d\Omega)_{i+r}$ fall "close" to the maxima and minima of $P_4(\cos\theta)$, plotted in Fig. 1(c).[15] (When the spin-flip and spin-nonflip amplitudes are out of phase, the positions of the maxima, minima, and zeros change with energy.)

We find aesthetically pleasing the idea that all secondary peaks in reactions of the type $P+B\to P+B$ and $P+B\to P'+B'$ (P and P', pseudoscalar mesons; B and B', spin-$\frac{1}{2}$ baryons) are resonance effects and wish to propose simple experiments to test this conjecture.[16] We propose measurements of the differential cross sections at energies W, $W_R-\frac{1}{2}\Gamma$, and $W_R+\frac{1}{2}\Gamma$ in regions where significant enhancements in the cross sections have been observed. Polarization measurements at the same energies would be also valuable.[17] In particular, an experimental study of this kind in the reaction $\pi^-+p\to\pi^0+n$ would be crucial, since it is a current idea that the near-forward dip of the differential cross section is due to the passage through a zero of the ρ Regge trajectory.[18]

There still remains the task of determining the phase and magnitude of the background amplitudes as functions of the angle and to fully justify our assumptions. This could be done, we believe, by assuming different admixtures of resonant states and determining the background amplitudes, magnitudes, and phases at each angle from $(d\sigma/d\Omega)_i$ and $[P(d\sigma/d\Omega)]_i$ until (hopefully) consistency with $(d\sigma/d\Omega)_b$

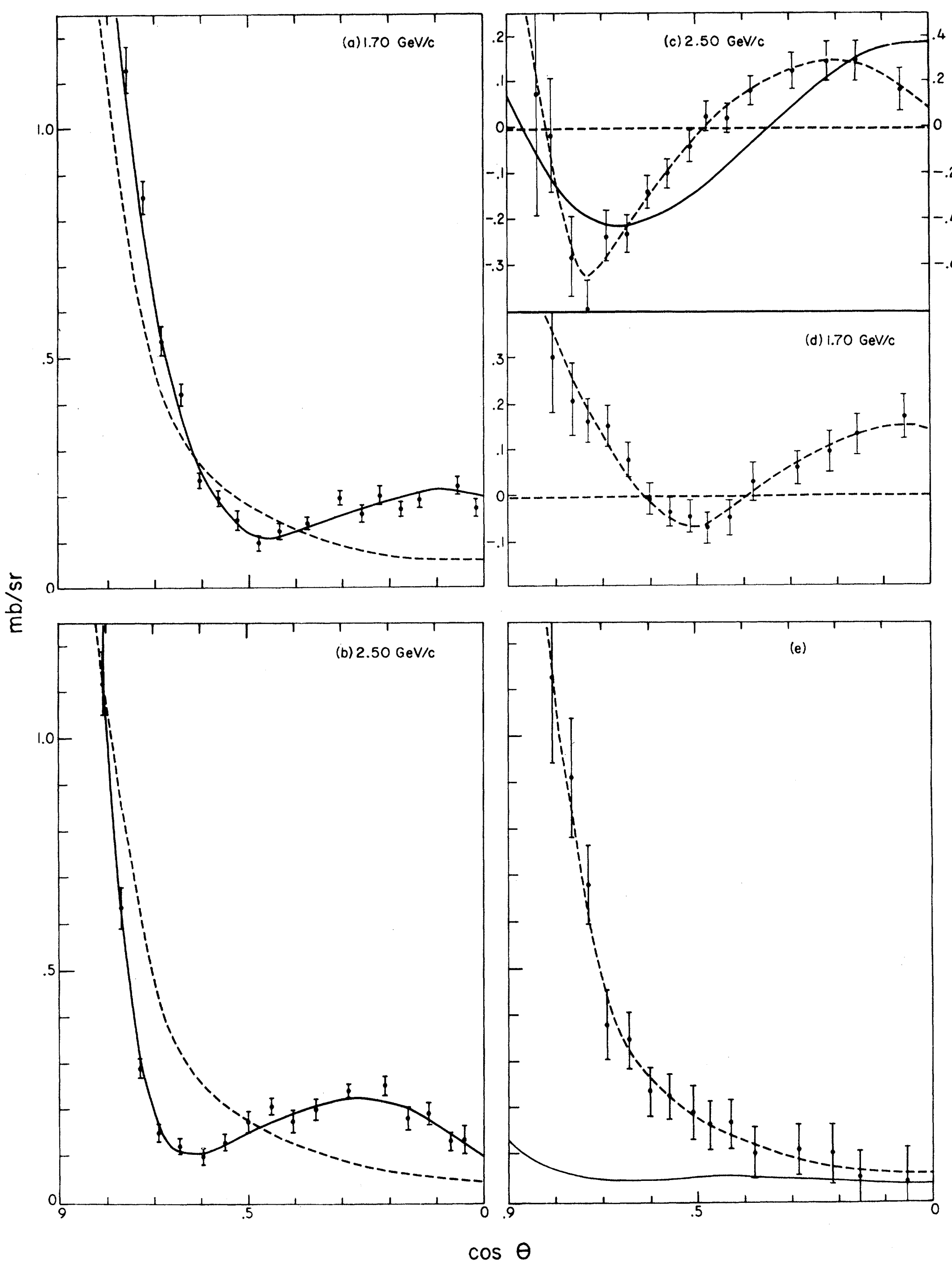

FIG. 1. Plots of $(k^2/k^2{}_R)(d\sigma/d\Omega)$ at (a) 1.7 BeV/c, (b) 2.5 BeV/c; $(k^2/k^2{}_R)(d\sigma/d\Omega)_{i+r}$ at (c) 2.5 BeV/c, (d) 1.7 BeV/c; and (e) $(k^2/k^2{}_R)(d\sigma/d\Omega)_b$. ($k_R$ is the c.m. momentum at the resonance energy.) The dashed lines in (c), (d), and (e) are freehand smooth curves for the above mentioned quantities. Smooth (dashed) lines for $(k^2/k^2{}_R)(d\sigma/d\Omega)_b$ have been also drawn in (a) and (b) for easier comparison. The continuous lines in (a) and (b) have been obtained by adding the curves for $(k^2/k^2{}_R)(d\sigma/d\Omega/_b$ and $(k^2/k^2{}_R)(d\sigma/d\Omega)_{i+r}$ at the corresponding energy. The continuous line in (e) is the contribution to $(k^2/k^2{}_R)(d\sigma/d\Omega)_r$ at 1.7 and 2.5 BeV/c from a $G_{7/2}$ resonance of elasticity $x = 0.27$. The continuous line in (e) (right-hand scale) is the Legendre polynomial $P_4(\cos\theta)$. We have used in all our calculations data from Ref. 5.

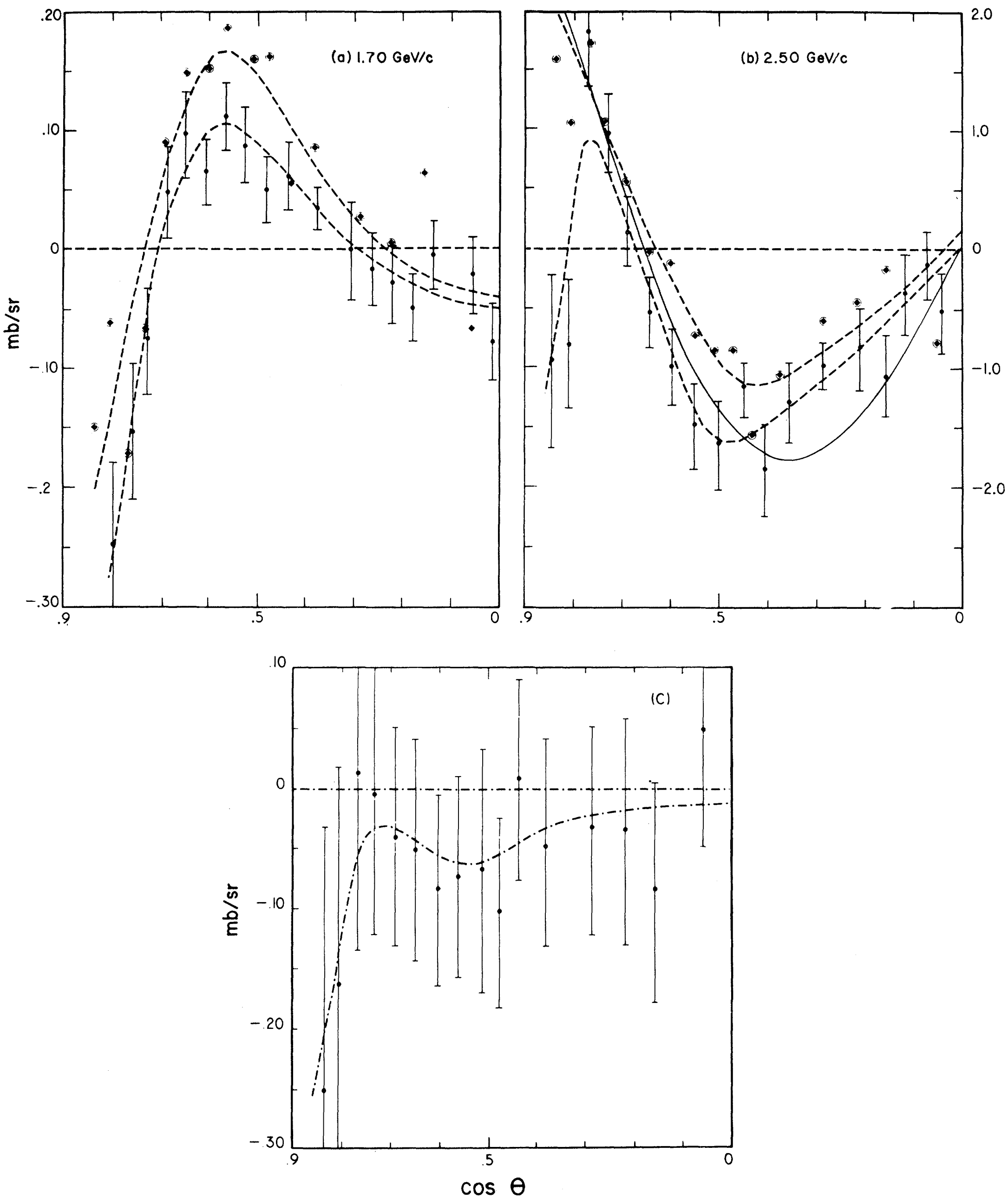

FIG. 2. Plots of $(k^2/k^2{}_R)(Pd\sigma/d\Omega)$ (points with error bars) and $(k^2/k^2{}_R)(Pd\sigma/d\Omega)_i$ at (a) 1.7 BeV/c and (b) 2.5 BeV/c. The error bars for $(k^2/k^2{}_R)(Pd\sigma/d\Omega)_i$ (not shown in the figure) are roughly twice as large as those for $(k^2/k^2{}_R)(Pd\sigma/d\Omega)$. We have plotted in (c) $(k^2/k^2{}_R(Pd\sigma/d\Omega)_b$. The lower dashed lines in (a) and (b) are smooth curves for $(k^2/k^2{}_R(Pd\sigma/d\Omega)$ [which are identical to the best fits of Yokosawa et al. (Ref. 5) in the region $0.9 \leq \cos\theta \leq 0.4$]; the upper dashed lines are smooth curves for $(k^2/k^2{}_R(Pd\sigma/d\Omega)_i$. The dot-dash line in (c) is a smooth curve for $(k^2/k^2{}_R)(Pd\sigma/d\Omega)_b$ (which obviously should not be taken very seriously). The smooth curves for $(k^2/k^2{}_R)(Pd\sigma/d\Omega)_i$ and $(k^2/k^2{}_R)(Pd\sigma/d\Omega)_b$ have been obtained using as our input smooth lines for $(k^2/k^2{}_R)(Pd\sigma/d\Omega)$ at the appropriate energies (see text). Finally, the continuous curve in (b) (right-hand scale) represents the function $-P_4'(\theta)$.

and $[P(d\sigma/d\Omega)]_b$ [as well as with $d\sigma/d\Omega$ and $P(d\sigma/d\Omega)$] at all angles is achieved.[19,20] We are making preliminary calculations assuming a pure $G_{7/2}$ resonance.

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[1]S. C. Frautschi, Phys. Rev. Letters 17, 772 (1966).

[2]G. Chew, Phys. Rev. Letters 16, 60 (1966).

[3]If the t-channel helicity-flip amplitude vanishes, the polarization vanishes necessarily. The converse is not true however.

[4]G. T. Hoff, Phys. Rev. 154, 1331 (1967).

[5]We have used the data of R. J. Sterling, R. E. Hill, N. E. Booth, S. Suwa, and A. Yokosawa, Enrico Fermi Institute for Nuclear Studies Report No. EFINS 66-29. Part of these experimental results were given in S. Suwa et al., Phys. Rev. Letters 15, 560 (1965); and A. Yokosawa et al., Phys. Rev. Letters 16, 714 (1966).

[6]The resonance origin of the secondary diffraction peak was suggested a while ago by D. E. Damouth, L. W. Jones, and M. I. Perl, Phys. Rev. Letters 11, 287 (1963).

[7]See, for instance, R. H. Dalitz, Ann. Rev. Nucl. Sci. 13, 339 (1963).

[8]The idea is to assign the same phase to all these resonant amplitudes.

[9]Provided other assumptions we are going to introduce later are valid.

[10]These equations are special cases of more general relations obtained by means of the substitution $\pi/4 \rightarrow \delta_0$, $3\pi/4 \rightarrow \delta_0 + \pi/2$, where δ_0 is less than $\pi/2$. In the derivation of these equations it is assumed that the conditions stated in assumptions (1) and (2) hold in the interval $\delta_0 \leq \delta \leq \delta_0 + \pi/2$.

[11]We are using values for W_R and Γ obtained from the total cross section and relating δ to the energy by means of Eq. (1). While the partial-wave analysis of Yokosawa et al. gives the same value for W_R, their resonance eigenphase, as deduced from their Argand plot, does not have the expected values $\pi/4$ and $3\pi/4$ at 1.7 and 2.5 BeV/c, respectively. The phases at 1.7 BeV/c (δ_l) and 2.5 BeV/c (δ_h), however, satisfy quite well the relationship $\delta_h = \delta_l + \pi/2$. According to Ref. 10 our results should still be valid.

[12]We have subtracted and added directly experimental points located within a $\cos\theta$ interval of 0.02 at most; otherwise we have made linear interpolations. Interpolations have been made only between experimental points separated by less than 0.05.

[13]The detailed shape of the curve for $[P(d\sigma/d\Omega)]_b$ should not be taken very seriously because of the large fluctuations. It should be noticed that the slight oscillatory behavior it shows, if actually present, is not inconsistent with the nonoscillatory behavior of $(d\sigma/d\Omega)_b$. (It could come from a variation of the relative phase, $\alpha - \beta$, with angle.)

[14]It should be noticed that the expected behavior is the opposite for a $G_{9/2}$ resonance.

[15]$(d\sigma/d\Omega)_r$, plotted in Fig. 1(e), does not change very much with angle in the region $0.8 \leq \cos\theta \leq 0$.

[16]We might extend our conjecture to phenomenologically different reactions.

[17]Equations (6) and (7) are valid for reactions of the type $P + B \rightarrow P' + B'$ in general, and charge exchange scattering in particular. Equations (4) and (5) are also valid provided we substitute for the elasticity x, the product $\pm(x_i x_f)^{1/2} = \pm(\Gamma_i \Gamma_f/\Gamma^2)^{1/2}$, where Γ_i (Γ_f) is the initial (final) partial width. When the sign is negative the phase of the partial-wave amplitude differs by π from the eigenphase δ.

[18]P. Sonderegger et al., Phys. Letters 20, 75 (1966); G. Höhler et al., Phys. Letters 20, 79 (1966).

[19]$(d\sigma/d\Omega)_i$ and $P(d\sigma/d\Omega)_i$ at any given angle or more two energies determine without any ambiguity, except for experimental errors, the phase and magnitude of a_b and b_b at this angle, once a certain admixture of resonance contributions have been assumed.

[21]We are incorporating the data at 1.88 and 2.27 BeV/c.